Damping criteria for thermal acoustic oscillations in slush and liquid hydrogen systems*

Y.F. Gu and K.D. Timmerhaus

Department of Chemical Engineering, University of Colorado, Boulder, CO 80309-0424, USA

Thermal acoustic oscillations in cryogenic systems are generally highly undesirable since large amounts of heat can be transferred into these systems during such oscillations. Further, the vibrations in these systems due to thermal oscillations can cause measurement and even structural difficulties. Considerable effort has been devoted in developing techniques for damping such oscillations by attaching resonators, inserting wires, etc. A theoretical analysis of these damping techniques has recently been completed. In this study a number of the damping criteria were investigated. These damping criteria were developed by investigating the stability characteristics obtained in an earlier analysis. This study indicates that thermal acoustic oscillations can be damped effectively by a number of approaches such as changing the tube radius, adjusting the length ratio of the warm section to the cold section and varying the temperature ratio or temperature profile along the tube.

Keywords: thermal acoustic oscillation; damping; hydrogen

Thermal acoustic oscillations are oscillations initiated by a large temperature gradient along a tube closed at one end and inserted into a cryogenic system. A theoretical analysis of such oscillations is complicated owing to the large temperature gradient and associated large variations in fluid properties along the length of the tube. The stability aspects of such oscillations in liquid hydrogen systems have been investigated in previous studies[1-4]. By analysing these results, several criteria for damping thermal acoustic oscillations have been ascertained, including the selection of a suitable tube radius, adjusting the ratio of the tube lengths in the warm and cold sections of the tube and possibly changing the temperature ratio or gradient along the tube. By utilizing one or more of these damping criteria, thermal acoustic oscillations can probably be eliminated in the design stage of cryogenic systems.

Earlier work has shown that there are two types of resistance encountered in oscillating flow, namely viscous and inertial resistances. Further, the driving force for initiating thermal acoustic oscillations is presumably caused by the large temperature gradient that exists along the oscillating tube. If one accepts these conclusions, it is clear that oscillations can be damped if the resistances are sufficiently large to restrict the flow of the fluid in the tube. On the other hand, thermal acoustic oscillations can also be damped if the driving force for such oscillations can be reduced to a value that is less than the summation of these two resistances. The quantitative analysis of such damping methods has recently been evaluated by theoretically analysing the stability characteristics of thermal acoustic oscillations when subjected to various restraints, and some of the results are presented here.

Damping effect of changing tube radius

As noted above, thermal acoustic oscillations can be damped by increasing either the viscous force or the inertial force, i.e. by increasing the resistances in the oscillating tube. Historical evidence has shown that viscous resistance is prominent in the momentum boundary layer close to a solid surface. From this it is clear that the smaller the tube radius, the larger is the viscous resistance in an oscillating tube. Therefore, oscillations may often be damped simply by reducing the tube radius, viz. by increasing the viscous resistance in the tube. This conclusion is supported by an inspection of the left-hand branches of the stability curves in *Figures 1* and *2* where no thermal acoustic oscillations are predicted to occur if the tube radius is less than a specified value for a fixed temperature ratio α and length ratio ξ. Here, α is the ratio of the temperature at the warm end of the oscillating tube to the temperature at the

---

*Paper presented at the 1991 Space Cryogenics Workshop, 18–20 June 1991, Cleveland, OH, USA

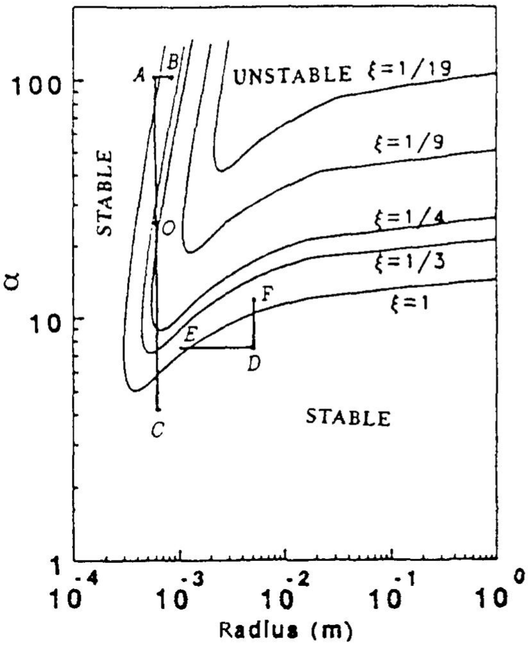

Figure 1 Critical radii for thermal acoustic oscillations in a normal boiling point parahydrogen system for a tube length of 1 m and $\xi \leq 1$. A step temperature profile has been assumed with the open end of the tube located above the liquid surface

cold end of the tube, and ξ is the ratio of the tube length in the warm section of the tube to that in the cold section of the tube. For example, the occurrence of thermal acoustic oscillations is predicted when the tube radius is r_B (see *Figure 1*) and the length ratio ξ is unity. However, no oscillations will occur if the tube radius is reduced to r_A. The curves in *Figures 1* and *2* were obtained by analysing the stability characteristics for thermal acoustic oscillations in a tube closed at the warm end with the open end located above the liquid surface. In these calculations a step temperature profile was assumed to exist along the length of the tube.

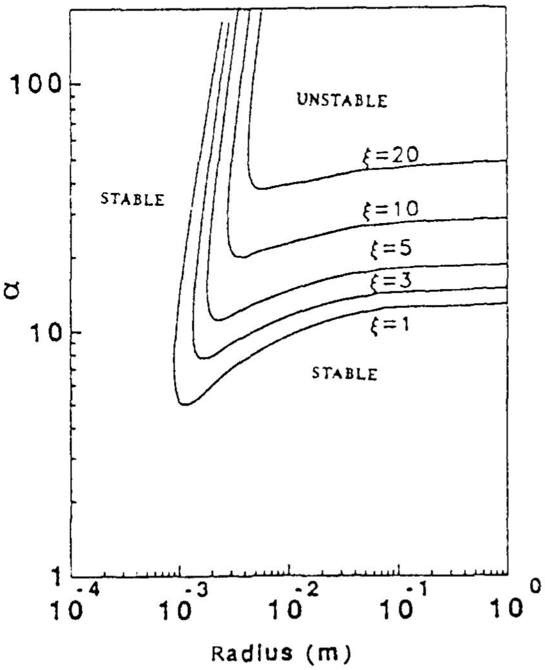

Figure 2 Critical radii for thermal acoustic oscillations in a normal boiling point parahydrogen system for a tube length of 1 m and $\xi \geq 1$. A step temperature profile has been assumed with the open end of the tube located above the liquid surface

It should be noted that the tube radius for the warm section of the oscillating tube has a larger effect on the oscillations than the tube radius in the cold section. This is attributed to the large variation in kinematic viscosity of the gas in the tube when proceeding from the warm temperature to the cold temperature. For example, the kinematic viscosity ν for parahydrogen vapour at ambient pressure and temperature (300 K) is 1.094×10^{-4} m^2 s^{-1}. However, it is 8.43×10^{-7} m^2 s^{-1} at the normal boiling point (NBP) (20.268 K) of hydrogen[5]. This significant difference in the value of kinematic viscosity results in a boundary layer in the 300 K section of the tube that is eleven times thicker than that found in the 20.268 K section of the tube. Since the viscous resistance in the warm section is considerably larger than that in the cold section, it becomes evident that a change in tube radius for the warm section will be more effective in damping the thermal oscillations than a change in tube radius for the cold section.

Thermal acoustic oscillations may also be damped by increasing the inertial force in the oscillating flow, i.e. by increasing the mass of gas undergoing oscillation. It is apparent that the mass of gas in an oscillating tube can be increased by increasing the tube radius. This is confirmed with the existence of a right-hand branch for each stability curve as shown in *Figures 1* and *2*. For example, thermal acoustic oscillations are predicted to occur at point E in *Figure 1* when the length ratio ξ is equal to unity. They may be damped if the tube radius is increased to r_D. However, the required tube radius to achieve such damping in hydrogen may attain such large proportions as to be impractical. Further, an asymptote is predicted to exist for the right-hand branch of each stability curve. The asymptote indicates that increasing the tube radius actually may have little effect in damping the oscillations if the temperature ratio along the tube is less than but close to this asymptotic value. Obviously, thermal acoustic oscillations cannot be damped by increasing the tube radius if the temperature ratio is greater than this asymptotic value. No such asymptotic characteristics for the stability curves have been predicted for thermal acoustic oscillations in helium systems[6]. These asymptotic characteristics in hydrogen may be due to the extremely low density of hydrogen vapour, which, in turn, results in a minimal increase in the inertial force even with a large increase in tube radius. Additionally, the assumptions of negligible radial pressure and temperature variations used in predicting these stability curves may be inadequate when the tube radius is relatively large. Experimental verifications are needed to investigate the predicted asymptotic characteristics of thermal acoustic oscillations in hydrogen systems.

Thermal acoustic oscillations have been initiated with greater ease when the open end of the tube is located below the liquid surface. *Figure 3* depicts the stability curves for such gas–liquid oscillations in a triple-point (TP) parahydrogen system. The lowest temperature ratio for initiating thermal acoustic oscillations is obtained when the length ratio ξ approaches zero. The oscillation frequencies under such conditions of submersion are much lower than when the open end of the tube

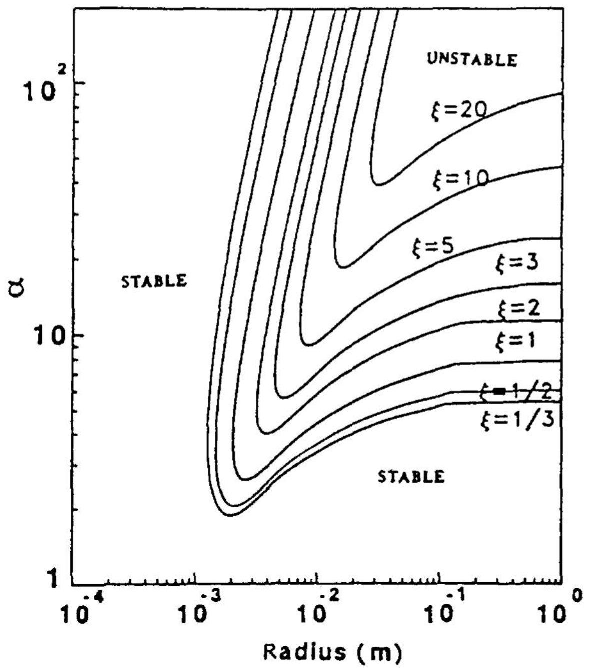

Figure 3 Critical radii for the thermal acoustic oscillations in a triple-point parahydrogen system when the open end of the tube is located below the liquid surface. A step temperature profile has been assumed. Total tube length is 1.1 m with 0.1 m located below the liquid level

is located above the liquid surface, since the liquid column in the tube is also oscillating. Note that if the open end of the tube is located below a slush hydrogen surface, the oscillation frequency will be appreciably lower than that predicted when the open end is submerged below a liquid hydrogen surface, since slush hydrogen exhibits a higher density.

A smaller temperature ratio is required for initiating thermal acoustic oscillations when the open end of the tube is located below the liquid level because pressure build-up in the oscillating tube is more rapid when the open end of the oscillating tube is sealed by a liquid column. A lower oscillation frequency results because oscillation of the liquid column is driven by density waves whose speed approaches that of the macro mass flow velocity. However, the oscillation speed is much higher in an oscillation tube when its open end is located above the liquid level since the acoustic waves travel at speeds approaching the local sonic velocity.

The minimum tube radius for initiating thermal acoustic oscillations is ≈1 mm (*Figure 3*). It should be noted that this result is for triple-point parahydrogen, where the system pressure is maintained at 52 mmHg (6.93 kPa). However, if slush hydrogen maintained at its triple-point temperature is transferred at ambient pressure, the kinematic viscosity will be approximately one-eighth of that at the triple-point pressure since the density of the hydrogen vapour increases with increase in pressure. Therefore, the minimum tube radius required for damping such oscillations must be less than $1/8^{1/2}$, or less than 0.375 mm. This may be too small a radius for practical use. However, this indicates that thermal acoustic oscillations in a slush hydrogen system may be initiated even in a tube with a very small tube radius. This has been experimentally confirmed in the NIST slush hydrogen facility, where thermal acoustic oscillations have been observed in a tube with a radius of 0.6 mm[7,8].

Damping effect of changing temperature ratio

The driving force for thermal acoustic oscillations is attributed to the large temperature gradient existing along the oscillating tube. Past experience has shown that thermal acoustic oscillations can be damped if this driving force is low or the oscillatory resistance is large. A decrease in the temperature ratio reduces the driving force that is available for thermal acoustic oscillations, and if decreased below a certain value may damp the thermal acoustic oscillations. The minimum temperature ratio for initiating thermal acoustic oscillations is ≈5 when a tube is inserted into a parahydrogen system and the open end located above the liquid surface. This is equivalent to a temperature of 100 K for NBP parahydrogen or 70 K for triple-point (TP) hydrogen. Apparently, thermal acoustic oscillations can be totally avoided if the temperature of the warm section of the inserted tube is maintained below these temperatures. However, the minimum temperature ratio required for eliminating oscillations is considerably lower when the open end of the tube is located below the liquid level (see *Figure 3*).

It is interesting that thermal acoustic oscillations can occasionally also be damped by increasing the temperature ratio. This is demonstrated in *Figure 1*, where thermal acoustic oscillations are damped when the temperature ratio is increased from point O to point A. This damping is attributed to the rapid change in viscous resistance that accompanies an increase in temperature, since the viscosity varies with temperature according to a power relationship expressed by

$$\nu = \mu/\rho \sim T^{1+\beta} \qquad (1)$$

where μ is the viscosity of the fluid, ρ is the density and β is a regressed constant. The value of β is usually greater than zero (0.7511 for parahydrogen[1] and 0.647 for helium[6]). Thus, even though an increase in the value of α increases the energy transferred into the oscillation system, thereby increasing the driving force of the oscillations, the viscous resistance increases more rapidly in the warm section of the tube because of the rapid rise in the value of the kinematic viscosity for the gas. If the resistance is increased to such an extent that the gas becomes relatively immobile in the tube, the thermal acoustic oscillations can often be damped. Since the viscous effect is only predominant in the momentum boundary layer, this damping can only be achieved when the tube radius is relatively small. This condition does not exist when the tube radius is relatively large.

Damping effect by altering length ratio ξ

A change in the tube length ratio of the warm section of the tube to the length in the cold section of the tube is another approach investigated for damping thermal acoustic oscillations. This is clearly demonstrated in *Figures 1*, *2* and *3*.

Generally, stability characteristics of thermal acoustic oscillations are determined from the three different forces existing in the oscillation system, i.e. the driving force, viscous resistance and inertial force. The driving force is affected by the temperature ratio and gradient along the tube and also by the heat transfer area (i.e. the tube length and radius in the warm section of the tube). The inertial force is affected by the oscillating mass in the tube which, in turn, is directly related to the mean temperature of the fluid, the system pressure, the length ratio and the tube radius. The viscous resistance, on the other hand, is determined by the viscosity of the fluid, particularly the viscosity in the warm section of the tube, the velocity distribution, the length ratio and the tube radius. It should be noted that the non-linear relationship between the viscous resistance and the length ratio is critical in determining the effect of the length ratio on the stability characteristics since the viscous resistance is related to the velocity of the fluid and the latter is zero at the warm closed end of the tube and reaches its maximum value at the cold open end of the tube.

When the open end of the tube is located above the liquid level, the inertial force will be affected by the length ratio of the tube, since the mean density of the gas in the tube is inversely proportional to the mean temperature along the tube. The magnitude of the driving force is proportional to the tube length in the warm section because it is directly related to the heat transfer area. However, the resistance is affected not only by the mean temperature but also by the tube length in the warm section of tube. When the length ratio ξ is less than unity, the inertial force is larger owing to the longer tube length in the cold section while the driving force for oscillations is smaller owing to the shorter tube length in the warm section. These two factors create an environment that requires a higher temperature ratio for initiating thermal acoustic oscillations in the relatively low temperature ratio range (see line O−C in *Figure 1*) and a lower temperature ratio for damping such oscillations in the relatively high temperature ratio range (see line O−A in *Figure 1*), even with a decrease in the viscous resistance. When the length ratio ξ is greater than unity, the viscous resistance dominates the oscillation because of the viscosity versus temperature relationship and the relatively longer tube length in the warm section. Under these conditions, the contribution by the cold section to the inertial force will be insignificant and the overall effect will be similar to that obtained when ξ is less than unity. Hence the unstable region for the stability curves of thermal acoustic oscillations will be reduced whenever ξ is either greater or smaller than unity.

However, when the open end of the tube is located below the liquid level, the inertial force is greatly increased owing to the introduction of the liquid column in the tube. Thus, when the length ratio ξ is less than unity, the change in the inertial force in the cold section of the tube will be insignificant if the liquid column height is kept constant. From this, one can conclude that the decrease in the viscous resistance for the cold section of tube should dictate the oscillation characteristics owing to the viscosity vs temperature power law, i.e. a smaller value in length ratio ξ will result in a broader unstable region for the stability curve associated with thermal acoustic oscillations. This is consistent with the results shown in *Figure 3*. However, when the value of ξ is greater than unity, the stability characteristics resemble those when the open end is located above the liquid level, since the inertial resistance involved in the oscillation remains fairly constant regardless of whether the open end of the oscillating tube is located above or below the liquid surface (see *Figure 3*).

Note that a smaller temperature ratio is necessary for initiating thermal acoustic oscillations when the open end of the tube is below the liquid surface. This is attributed to the rapid build-up of pressure in the small volume of the gas confined in the sealed tube. Additionally, a lower value of the oscillation frequency results in a smaller fluid velocity of oscillation and also a thicker acoustic momentum boundary layer (the latter is proportional to $(\nu/\omega)^{1/2}$, where ω is the oscillation frequency). Both of these effects can reduce the velocity gradient of the fluid close to the wall and thereby decrease the viscous resistance of oscillation. In other words, a smaller temperature ratio is required for initiating thermal acoustic oscillations when the open end of the tube is located below than when it is located above the liquid surface.

Damping effect of modifying temperature gradient

The results in *Figures 1* and *2* were obtained using a step temperature profile along the oscillating tube (see curve A in *Figure 4*). A recent study investigating the effect of different temperature profiles on thermal acoustic oscillations revealed that the temperature profile does affect the stability of thermal acoustic oscillations[3]. The different temperature profiles considered in this study are shown in *Figure 4* and the stability characteristics of thermal acoustic oscillations for these temperature profiles are shown in *Figure 5*. Conclusions drawn from the numerical results presented in these figures included the following:

1 As the temperature profiles along the tube become steeper, thermal acoustic oscillations can be initiated at continuously decreasing smaller minimum temperature ratios.
2 An inspection of the right-hand branch of the stability curve indicates that thermal acoustic oscillations can be initiated more easily whenever a steeper temperature profile exists along the tube. This condi-

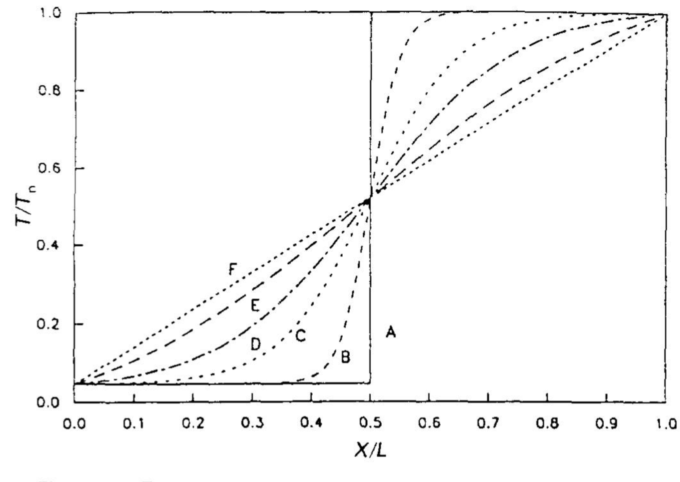

Figure 4 Temperature profiles for stability curves displayed in *Figure 5*

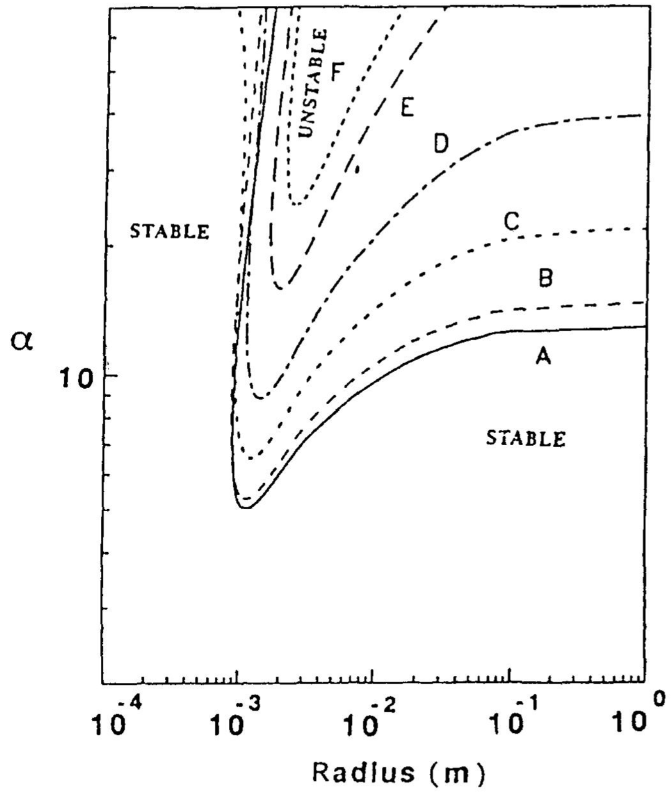

Figure 5 Effect of temperature profile on stability characteristics

tion is attributed to the higher driving forces for oscillations that are present with the steeper temperature profiles. This is mainly due to the larger temperature difference for heat transfer, since there is little change in the inertial force as long as the mean temperature along the tube is relatively unchanged.

3 For the left-hand branch of the stability curve, the effect of the temperature profile along the tube on the thermal acoustic oscillations is more complex. For example, the left-hand branches of curves B and C in *Figure 5* are located to the left of curve A, which is based on a step temperature profile along the tube. This unusual behaviour is a consequence of the different effects that the driving force, viscous resistance and inertial force have on the oscillation characteristics with a change in the temperature profile along the tube. However, note that the unstable region decreases quickly as the temperature profile approaches a linear distribution.

Conclusions

Different methods of damping thermal acoustic oscillations in hydrogen systems have been analysed in this study. Conclusions drawn from these results indicate that:

1 Theoretically, thermal acoustic oscillations can be damped by either reducing or increasing the radius of the tube, i.e. by increasing either the viscous or inertial resistance in the oscillation. However, increasing the tube radius will only provide the desired damping if the temperature ratio for the system is lower than the asymptotic value of the right-hand stability curve.

2 Thermal acoustic oscillations may be damped by changing the temperature ratio along the tube. A minimum temperature ratio for initiating thermal acoustic oscillations in both normal boiling point and triple-point parahydrogen systems is ≈ 5 when the open end of the tube is located above the liquid surface. However, thermal acoustic oscillations can be initiated more readily when the open end of the tube is located below the liquid surface. A lower temperature ratio, however, is required for damping the oscillations generated under these conditions. It should also be noted that thermal acoustic oscillations may be damped by increasing the temperature ratio along the tube, i.e. increasing the viscous resistance in the warm section of the oscillating tube.

3 Changing the length ratio of the warm section to the cold section is another option for damping thermal acoustic oscillations. A length ratio of unity provides the largest unstable region for initiating thermal acoustic oscillation when the open end of the tube is located above the liquid level. For oscillation systems in which the open end of the tube is located below the liquid surface, larger length ratios contribute to the damping of thermal acoustic oscillations since the inertial force in the cold section is insensitive to changes in the length ratio ξ.

4 The addition of thermal insulation along the tube appears to be useful for damping thermal acoustic oscillations, particularly when the temperature profile along the tube approaches a linear distribution.

Acknowledgements

The authors express their deep appreciation for the assistance of R. DeWitt, technical officer for this NASA study. They thank T.M. Flynn for his kind help and encouragement. Appreciation is due P. Ludtke and O. Voth of NIST for their helpful advice.

References

1 **Gu, Y.F. and Timmerhaus, K.D.** Thermal acoustic oscillations in normal-boiling point liquid hydrogen systems *NASP Contractor Report* (1990)

2 **Gu, Y.F. and Timmerhaus, K.D.** Thermal acoustic oscillations in triple point liquid hydrogen systems *NASP Contractor Report* (1990)

3 **Gu, Y.F. and Timmerhaus, K.D.** Effect of tube temperature profile on thermal acoustic oscillations in liquid hydrogen systems, in: *Proceedings of the Sixth International Cryocooler Conference, Plymouth, MA* (1990) 329–340

4 **Gu, Y.F. and Timmerhaus, K.D.** Thermal acoustic oscillations in liquid hydrogen systems, paper presented at the 8th Intersociety Cryogenic Symposium, AIChE Annual Meeting, Houston, TX, April, 1991

5 **McCarty, R.D., Hord, J. and Roder, H.M.** Selected properties of hydrogen (engineering design data), *NBS Monograph* 168 (1981)

6 **Rott, N.** Thermally driven acoustic oscillations, part 2: stability limit for helium, *J Appl Math Phys (ZAMP)* (1973) **24** 54–72

7 **Daney, D.E., Ludtke, P.R., Sindt, C.F. and Chelton, D.B.** Slush hydrogen fluid characterization and instrumentation analysis *NBS Report* No. 9701, NASA Project 31505-3150452 (1967)

8 **Ludtke, P.R.** Slush hydrogen flow facility, *NBS Report* No. 9752 (1969)

9 **Tward, E. and Mason, P.V.** Damping of thermalacoustic oscillations, *Adv Cryog Eng* (1983) **28** 807–815